10.7 Write and Graph Equations of Circles

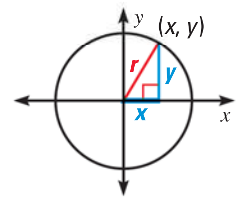
Before	You wrote equations of lines in the coordinate plane.
Now	You will write equations of circles in the coordinate plane.
Why?	So you can determine zones of a commuter system, as in Ex. 36.

Key Vocabulary
- **standard equation of a circle**

Let (x, y) represent any point on a circle with center at the origin and radius r. By the Pythagorean Theorem,

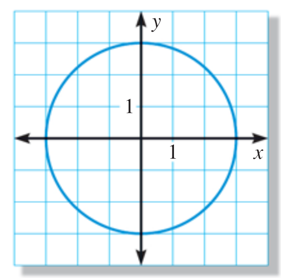
$$x^2 + y^2 = r^2.$$

This is the equation of a circle with radius r and center at the origin.



EXAMPLE 1 Write an equation of a circle

Write the equation of the circle shown.



Solution

The radius is 3 and the center is at the origin.

$x^2 + y^2 = r^2$ **Equation of circle**

$x^2 + y^2 = 3^2$ **Substitute.**

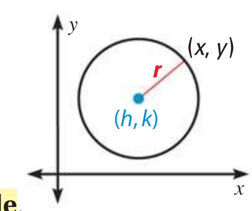
$x^2 + y^2 = 9$ **Simplify.**

▶ The equation of the circle is $x^2 + y^2 = 9$.

CIRCLES CENTERED AT (h, k) You can write the equation of *any* circle if you know its radius and the coordinates of its center.

Suppose a circle has radius r and center (h, k). Let (x, y) be a point on the circle. The distance between (x, y) and (h, k) is r, so by the Distance Formula

$$\sqrt{(x - h)^2 + (y - k)^2} = r.$$



Square both sides to find the **standard equation of a circle**.

KEY CONCEPT *For Your Notebook*

Standard Equation of a Circle

The standard equation of a circle with center (h, k) and radius r is:

$$(x - h)^2 + (y - k)^2 = r^2$$

EXAMPLE 2 Write the standard equation of a circle

Write the standard equation of a circle with center (0, −9) and radius 4.2.

Solution

$(x - h)^2 + (y - k)^2 = r^2$ — Standard equation of a circle

$(x - 0)^2 + (y - (-9))^2 = 4.2^2$ — Substitute.

$x^2 + (y + 9)^2 = 17.64$ — Simplify.

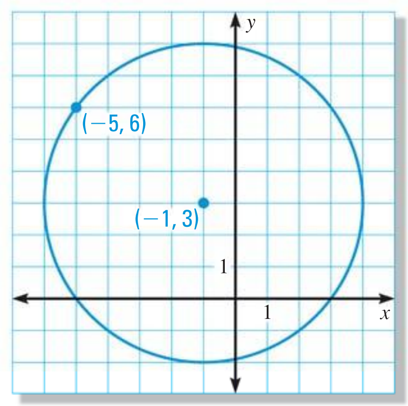
✓ GUIDED PRACTICE for Examples 1 and 2

Write the standard equation of the circle with the given center and radius.

1. Center (0, 0), radius 2.5
2. Center (−2, 5), radius 7

EXAMPLE 3 Write the standard equation of a circle

The point (−5, 6) is on a circle with center (−1, 3). Write the standard equation of the circle.



Solution

To write the standard equation, you need to know the values of *h*, *k*, and *r*. To find *r*, find the distance between the center and the point (−5, 6) on the circle.

$r = \sqrt{[-5 - (-1)]^2 + (6 - 3)^2}$ — Distance Formula

$= \sqrt{(-4)^2 + 3^2}$ — Simplify.

$= 5$ — Simplify.

Substitute $(h, k) = (-1, 3)$ and $r = 5$ into the standard equation of a circle.

$(x - h)^2 + (y - k)^2 = r^2$ — Standard equation of a circle

$[x - (-1)]^2 + (y - 3)^2 = 5^2$ — Substitute.

$(x + 1)^2 + (y - 3)^2 = 25$ — Simplify.

▸ The standard equation of the circle is $(x + 1)^2 + (y - 3)^2 = 25$.

✓ GUIDED PRACTICE for Example 3

3. The point (3, 4) is on a circle whose center is (1, 4). Write the standard equation of the circle.
4. The point (−1, 2) is on a circle whose center is (2, 6). Write the standard equation of the circle.

EXAMPLE 4 Graph a circle

USE EQUATIONS
If you know the equation of a circle, you can graph the circle by identifying its center and radius.

The equation of a circle is $(x - 4)^2 + (y + 2)^2 = 36$. Graph the circle.

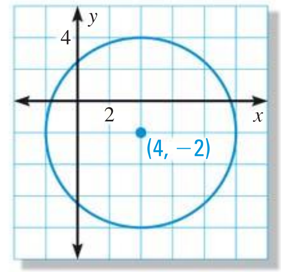
Solution

Rewrite the equation to find the center and radius.

$$(x - 4)^2 + (y + 2)^2 = 36$$

$$(x - 4)^2 + [y - (-2)]^2 = 6^2$$

The center is $(4, -2)$ and the radius is 6. Use a compass to graph the circle.



EXAMPLE 5 Use graphs of circles

EARTHQUAKES The epicenter of an earthquake is the point on Earth's surface directly above the earthquake's origin. A seismograph can be used to determine the distance to the epicenter of an earthquake. Seismographs are needed in three different places to locate an earthquake's epicenter.

Use the seismograph readings from locations A, B, and C to find the epicenter of an earthquake.

- The epicenter is 7 miles away from $A(-2, 2.5)$.
- The epicenter is 4 miles away from $B(4, 6)$.
- The epicenter is 5 miles away from $C(3, -2.5)$.

Solution

The set of all points equidistant from a given point is a circle, so the epicenter is located on each of the following circles.

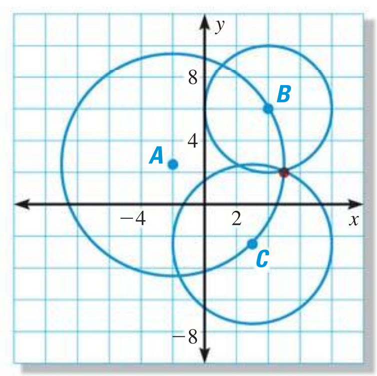
$\odot A$ with center $(-2, 2.5)$ and radius 7

$\odot B$ with center $(4, 6)$ and radius 4

$\odot C$ with center $(3, -2.5)$ and radius 5

To find the epicenter, graph the circles on a graph where units are measured in miles. Find the point of intersection of all three circles.

▶ The epicenter is at about $(5, 2)$.



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✓ GUIDED PRACTICE for Examples 4 and 5

5. The equation of a circle is $(x - 4)^2 + (y + 3)^2 = 16$. Graph the circle.

6. The equation of a circle is $(x + 8)^2 + (y + 5)^2 = 121$. Graph the circle.

7. Why are three seismographs needed to locate an earthquake's epicenter?

10.7 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 17, and 37

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 16, 26, and 42

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The standard equation of a circle can be written for any circle with known _?_ and _?_.

2. ★ **WRITING** *Explain* why the location of the center and one point on a circle is enough information to draw the rest of the circle.

EXAMPLES 1 and 2 on pp. 699–700 for Exs. 3–16

WRITING EQUATIONS **Write the standard equation of the circle.**

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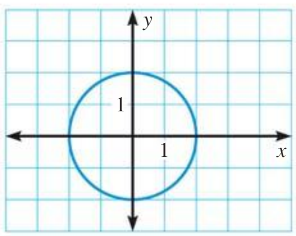

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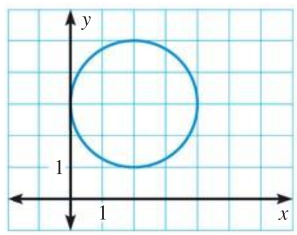

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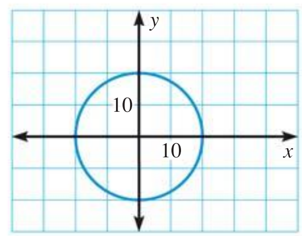

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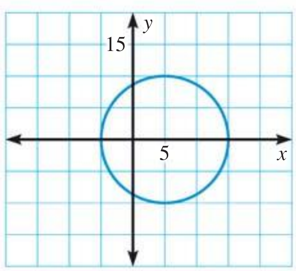

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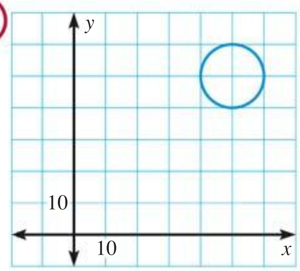
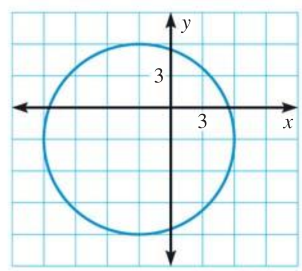


8.


WRITING EQUATIONS **Write the standard equation of the circle with the given center and radius.**

9. Center (0, 0), radius 7
10. Center (−4, 1), radius 1
11. Center (7, −6), radius 8
12. Center (4, 1), radius 5
13. Center (3, −5), radius 7
14. Center (−3, 4), radius 5

15. **ERROR ANALYSIS** *Describe* and correct the error in writing the equation of a circle.

> An equation of a circle with center (−3, −5) and radius 3 is $(x - 3)^2 + (y - 5)^2 = 9$.

16. ★ **MULTIPLE CHOICE** The standard equation of a circle is $(x - 2)^2 + (y + 1)^2 = 16$. What is the diameter of the circle?

Ⓐ 2 Ⓑ 4 Ⓒ 8 Ⓓ 16

EXAMPLE 3 on p. 700 for Exs. 17–19

WRITING EQUATIONS **Use the given information to write the standard equation of the circle.**

17. The center is (0, 0), and a point on the circle is (0, 6).
18. The center is (1, 2), and a point on the circle is (4, 2).
19. The center is (−3, 5), and a point on the circle is (1, 8).

EXAMPLE 4
on p. 701
for Exs. 20–25

GRAPHING CIRCLES **Graph the equation.**

20. $x^2 + y^2 = 49$

21. $(x - 3)^2 + y^2 = 16$

22. $x^2 + (y + 2)^2 = 36$

23. $(x - 4)^2 + (y - 1)^2 = 1$

24. $(x + 5)^2 + (y - 3)^2 = 9$

25. $(x + 2)^2 + (y + 6)^2 = 25$

26. ★ **MULTIPLE CHOICE** Which of the points does not lie on the circle described by the equation $(x + 2)^2 + (y - 4)^2 = 25$?

Ⓐ $(-2, -1)$ Ⓑ $(1, 8)$ Ⓒ $(3, 4)$ Ⓓ $(0, 5)$

ALGEBRA **Determine whether the given equation defines a circle. If the equation defines a circle, rewrite the equation in standard form.**

27. $x^2 + y^2 - 6y + 9 = 4$

28. $x^2 - 8x + 16 + y^2 + 2y + 4 = 25$

29. $x^2 + y^2 + 4y + 3 = 16$

30. $x^2 - 2x + 5 + y^2 = 81$

IDENTIFYING TYPES OF LINES **Use the given equations of a circle and a line to determine whether the line is a *tangent*, *secant*, *secant that contains a diameter*, or none of these.**

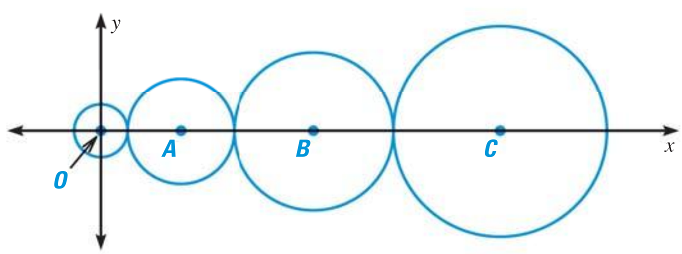
31. **Circle:** $(x - 4)^2 + (y - 3)^2 = 9$
Line: $y = -3x + 6$

32. **Circle:** $(x + 2)^2 + (y - 2)^2 = 16$
Line: $y = 2x - 4$

33. **Circle:** $(x - 5)^2 + (y + 1)^2 = 4$
Line: $y = \frac{1}{5}x - 3$

34. **Circle:** $(x + 3)^2 + (y - 6)^2 = 25$
Line: $y = -\frac{4}{3}x + 2$

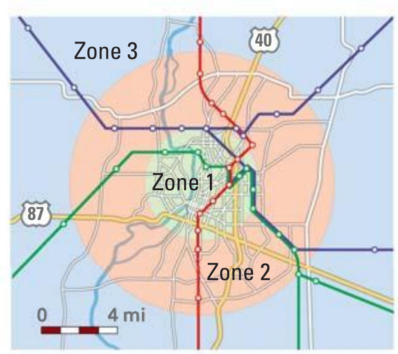
35. **CHALLENGE** Four tangent circles are centered on the x-axis. The radius of $\odot A$ is twice the radius of $\odot O$. The radius of $\odot B$ is three times the radius of $\odot O$. The radius of $\odot C$ is four times the radius of $\odot O$. All circles have integer radii and the point $(63, 16)$ is on $\odot C$. What is the equation of $\odot A$?



PROBLEM SOLVING

EXAMPLE 5
on p. 701
for Ex. 36

36. **COMMUTER TRAINS** A city's commuter system has three zones covering the regions described. Zone 1 covers people living within three miles of the city center. Zone 2 covers those between three and seven miles from the center, and Zone 3 covers those over seven miles from the center.



a. Graph this situation with the city center at the origin, where units are measured in miles.

b. Find which zone covers people living at $(3, 4)$, $(6, 5)$, $(1, 2)$, $(0, 3)$, and $(1, 6)$.

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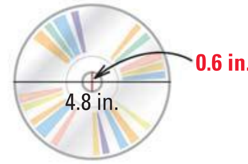
37. **COMPACT DISCS** The diameter of a CD is about 4.8 inches. The diameter of the hole in the center is about 0.6 inches. You place a CD on the coordinate plane with center at (0, 0). Write the equations for the outside edge of the disc and the edge of the hole in the center.



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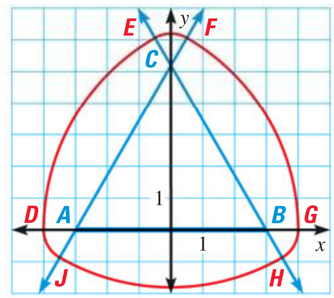
REULEAUX POLYGONS **In Exercises 38–41, use the following information.**

The figure at the right is called a *Reuleaux polygon*. It is not a true polygon because its sides are not straight. $\triangle ABC$ is equilateral.



38. $\overset{\frown}{JD}$ lies on a circle with center A and radius AD. Write an equation of this circle.

39. $\overset{\frown}{DE}$ lies on a circle with center B and radius BD. Write an equation of this circle.

40. **CONSTRUCTION** The remaining arcs of the polygon are constructed in the same way as $\overset{\frown}{JD}$ and $\overset{\frown}{DE}$ in Exercises 38 and 39. Construct a Reuleaux polygon on a piece of cardboard.

41. Cut out the Reuleaux polygon from Exercise 40. Roll it on its edge like a wheel and measure its height when it is in different orientations. *Explain* why a Reuleaux polygon is said to have constant width.

42. ★ **EXTENDED RESPONSE** Telecommunication towers can be used to transmit cellular phone calls. Towers have a range of about 3 km. A graph with units measured in kilometers shows towers at points (0, 0), (0, 5), and (6, 3).

a. Draw the graph and locate the towers. Are there any areas that may receive calls from more than one tower?

b. Suppose your home is located at (2, 6) and your school is at (2.5, 3). Can you use your cell phone at either or both of these locations?

c. City A is located at (−2, 2.5) and City B is at (5, 4). Each city has a radius of 1.5 km. Which city seems to have better cell phone coverage? *Explain*.

43. **REASONING** The lines $y = \frac{3}{4}x + 2$ and $y = -\frac{3}{4}x + 16$ are tangent to $\odot C$ at the points (4, 5) and (4, 13), respectively.

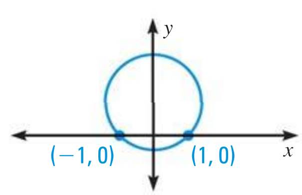
a. Find the coordinates of C and the radius of $\odot C$. *Explain* your steps.

b. Write the standard equation of $\odot C$ and draw its graph.

44. **PROOF** Write a proof.

GIVEN ▶ A circle passing through the points (−1, 0) and (1, 0)

PROVE ▶ The equation of the circle is $x^2 - 2yk + y^2 = 1$ with center at $(0, k)$.



◯ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

45. **CHALLENGE** The intersecting lines m and n are tangent to $\odot C$ at the points (8, 6) and (10, 8), respectively.

   a. What is the intersection point of m and n if the radius r of $\odot C$ is 2? What is their intersection point if r is 10? What do you notice about the two intersection points and the center C?

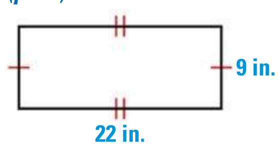
   b. Write the equation that describes the locus of intersection points of m and n for all possible values of r.

MIXED REVIEW

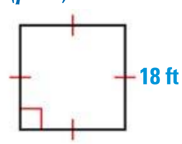
PREVIEW
Prepare for Lesson 11.1 in Exs. 46–48.

Find the perimeter of the figure.

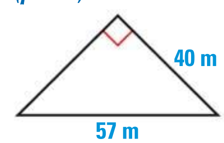
46. *(p. 49)*



47. *(p. 49)*



48. *(p. 433)*



Find the circumference of the circle with given radius r or diameter d. Use $\pi = 3.14$. *(p. 49)*

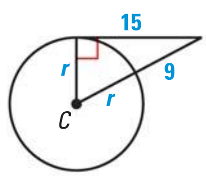
49. $r = 7$ cm

50. $d = 160$ in.

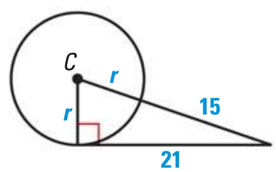
51. $d = 48$ yd

Find the radius r of $\odot C$. *(p. 651)*

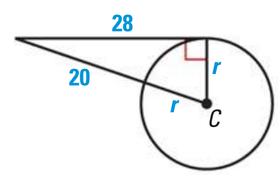
52.



53.



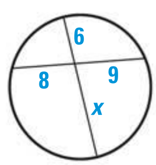
54.



QUIZ *for Lessons 10.6–10.7*

Find the value of x. *(p. 689)*

1.



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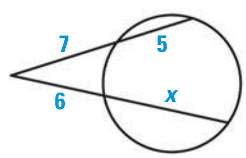
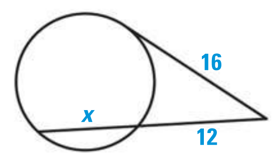


3.



In Exercises 4 and 5, use the given information to write the standard equation of the circle. *(p. 699)*

4. The center is (1, 4), and the radius is 6.

5. The center is (5, −7), and a point on the circle is (5, −3).

6. **TIRES** The diameter of a certain tire is 24.2 inches. The diameter of the rim in the center is 14 inches. Draw the tire in a coordinate plane with center at (−4, 3). Write the equations for the outer edge of the tire and for the rim where units are measured in inches. *(p. 699)*